2020 年度日本政府（文部科学省）奨学金留学生選考試験

QUALIFYING EXAMINATION FOR APPLICANTS FOR THE JAPANESE GOVERNMENT (MEXT) SCHOLARSHIP 2020

学科試験　　問題

EXAMINATION QUESTIONS

（学部留学生）

UNDERGRADUATE STUDENTS

数　　学　（Ｂ）

MATHEMATICS(B)

注意　☆試験時間は **60 分**。

PLEASE NOTE: THE TEST PERIOD IS **60 MINUTES**.

Nationality		No.	
Name	(Please print full name, underlining family name)	Marks	

Answer the following questions and fill in your responses in the corresponding boxes on the answer sheet.

1. Fill in the blanks with the correct numbers.

(1) The largest one among natural numbers that are less than

$$\log_2 3 \cdot \log_3 4 \cdot \log_4 5 \cdots \cdots \log_{2019} 2020$$

is $\boxed{}$.

(2) Let $f(x) = 1 + \dfrac{1}{x-1}$ $(x \neq 1)$. The solution of the equation $f(f(x)) = f(x)$ is $x = \boxed{① }$, $\boxed{② }$.

(3) Let a and b be real numbers with $b \geq 0$. When the equation $x^4 + ax^2 + b = 0$ has exactly two real solutions, the minimum value of $a + 2b$ is $\boxed{① }$, and the maximum value of $\lceil a - b \rceil$ is $\boxed{② }$. Here, $\lceil r \rceil$ denotes the smallest integer that is larger than or equal to the real number r.

(4) The division of a polynomial function $f(x)$ by $(x-1)^2$ gives the remainder $x+1$, and that by x^2 gives the remainder $2x+3$. Thus, the remainder of the division of $f(x)$ by $x^2(x-1)$ is

$$\boxed{① } x^2 + \boxed{② } x + \boxed{③ } .$$

(5) The angle θ $\left(0 < \theta < \dfrac{\pi}{2} \right)$ between the two lines $y = (2 - \sqrt{3})x$

and $y = (\sqrt{3} - 2)x$ on the xy-plane is [   ].

2. Cast a dice 3 times and let a, b, c be the number of dots on a side of the dice in order. Fill in the blanks with the answers to the following questions.

(1) What is the probability P_1 that there exists a triangle with the side lengths $\sqrt{a}$, $\sqrt{b}$, $\sqrt{c}$?

(2) Given that there exists a right triangle with the side lengths $\sqrt{a}$, $\sqrt{b}$, $\sqrt{c}$, what is the conditional probability P_2 that a, b, c are mutually different?

(3) When there exists a triangle with the side lengths $\sqrt{a}$, $\sqrt{b}$, $\sqrt{c}$ and one of the angles is 60 degrees, what is M, which is the maximum value of $\dfrac{bc}{a}$?

(1) $P_1 =$ [   ]

(2) $P_2 =$ [   ]

(3) $M =$ [   ]

3. Let C be a curve given by the parametric equations $x = \theta - \sin\theta$, $y = 1 - \cos\theta$ $(0 \le \theta \le 2\pi)$. Fill in the blanks with the answers to the following questions.

(1) Express $\dfrac{dy}{dx}$ in θ.

(2) Find the area S of the figure A, which is bounded by C and the x-axis.

(3) Find the volume V of the solid formed by rotation of A in (2) about the x-axis.

(1) $\dfrac{dy}{dx} = $

(2) $S = $

(3) $V = $